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Instability and collapse of waveguides on the water surface under the ice cover

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Abstract

We consider long gravity-flexural waves on a surface of a perfect liquid of a finite depth under elastic ice-plate. Such waves of small but finite amplitude are governed by the generalized Kadomtsev–Petviashvili (KP) equation, which contains higher spatial derivatives. The generalized KP equation admits waveguide solutions, describing waves periodic in the direction of propagation and localized in the transverse direction. Waveguide represents a wave being the nonlinear product of instability of carrier monochromatic wave with respect to transverse perturbations. Instability of waveguides in its nonlinear stage is studied. For this purpose we use the alternative description via the Davey–Stewartson (DS) equations for slowly varying amplitudes of monochromatic waves. The DS equations are asymptotically equivalent to the initial generalized KP equation. Behaviour of perturbations is determined by values of wavenumber of the carrier wave. We find, in particular, that for some range of wavenumbers of the carrier wave the waveguide is subjected to the local collapse which differs from collapse of waveguides in the fluid of infinite depth.

Keywords: Waveguide; Self-channelling; Local collapse

1. Introduction

Haragus-Courcelle and Il'ichev [1] derived the equation for long waves on the surface of water layer of finite depth in the presence of additional surface effects:

$$\partial_x\left(\partial_t\eta + \eta\partial_x\eta + \partial_x^3\eta + \partial_x^5\eta\right) + \partial_{yy}^2\eta = 0. \tag{1.1}$$

Here (t, x, y) are the non-dimensional time and Cartesian coordinates and $\eta = \eta(t, x, y)$ is the non-dimensional surface deviation. The surface effects in question are given either by gravity-capillary forces, or by forces and bending moments in a thin elastic plate, floating on the water surface. Under certain circumstances such a plate can model an ice cover (see, e.g., [2]).

In the case of gravity-capillary waves the Bond number $Bo = T/(\rho g H^2)$, where g is gravity, T is the coefficient of surface tension, ρ is the water density, H is the water depth, have to be close to the critical value $Bo^* = 1/3$ from below (in case when $Bo > Bo^*$ minus sign at the forth x-derivative in (1.1) must stay). The proximity of the Bond number to Bo^* implies the smallness of the characteristic spatial scales: for the water the depth has the order of several millimetres [3]. For such scales one must take viscous effects into consideration and consequently, the non-dissipative model has in fact a lack of physical meaning. Therefore, for the description of gravity-capillary waves Eq. (1.1) and its one-dimensional analogue – the Kawahara equation –

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have only formal character. Experimental verification of the Kawahara equation in the conditions of low gravity, where viscous effects are weakened, was considered by [3].

In the case of gravity-flexural waves the characteristic scales are much greater and influence of viscous effects may be neglected. The coefficient at the third power of wave number in the long wave expansion of the frequency is finite and the coefficient at the fifth power can take sufficiently large values, because of the large value of Young's module $E \sim 10^9\ \mathrm{N} \cdot \mathrm{m}^{-2}$. The condition that the terms with the third and fifth powers of wave number in the dispersion relation are of the same order reads

$$\frac{E}{12\rho(1-\sigma^2)} \sim \frac{g\lambda^2 H^2}{h^3},$$

where λ is the characteristic wave length, σ is Poisson's ratio and h is the thickness of the elastic plate. For example, if $h \sim 1$ m, $H \sim 10$ m, one has $\lambda \sim 100$ m and the characteristic wave amplitude about 1 m (see [1]). Therefore, there are no theoretical reasons visible to deny Eq. (1.1) as a possible model one for the description of long surface water waves under homogeneous elastic ice-cover, and we focus here our attention exactly on this application of (1.1).

Il'ichev [4] showed that Eq. (1.1) possesses waveguide type solutions (hereafter called waveguides), which in case of their stability realize propagation of energy without dispersive diffusion. In this sense these waves represent nonlinear structures which are analogous to plane solitary waves (solitons). The waveguides are the products of the transverse modulational instability (self-channelling) of carrier periodic waves.

By evident reasons there questions arise about the stability of waveguides and in case of their instability about what happens further if they are unstable. The answers are that the waveguides are always unstable, and that for some range of wavenumbers of the carrier monochromatic wave the waveguide is subjected to a local collapse (which differs from the collapse of waveguides in the fluid of infinite depth) and for the other range of wavenumbers it decays due to other mechanizms. We give here these answers using the alternative to (1.1) Davey–Stewartson (DS) equations which are asymptotically equivalent to (1.1), i.e., we derive from (1.1) the DS equations, describing slowly varying amplitudes of modulated wave packets, and furter perform the stability analysis for perturbations, obeying these equations. In this setting, the waveguide solution of (1.1) corresponds to a soliton solution of the DS equations. The problem is, thus, reduced to the question about instability of solitary wave, and the instability result, as concerns Schrödinger-type equations in the whole and the DS equations in particular, is well-known.

Zakharov and Rubenchik [5] (see also the survey by Kuznetsov et al., [6]) used the asymptotic method for investigation of the linear transverse instability of solitons of the Schrödinger equation with respect to long wave perturbations. Their method implies the construction of the unstable eigenfunctions basing on the neutrally stable eigenfunctions which can be written in the explicit form. It was found that soliton is always unstable, i.e. there exists unstable eigenfunction which is even for the elliptic Schrödinder equation an odd for hyperbolic one. Ablowitz and Segur [7] applied this method to the investigation of the transverse instability of solitons via the DS equations and also found that they are always unstable. Janssen and Rasmussen [8] found the threshold value of the unstable wavenumbers in the case of the elliptic Schrödinger equation. The unstable component of the dispersive curve for this case is given by Rypdal and Rasmussen [9], for example. Saffman and Yuen [10] numerically found the threshold of instability in the case of hyperbolic Schrödinger equation and also plotted the unstable component of the dispersive curve. A comprehensive review of the transverse instability of solitary waves was given by Kivshar and Pelinovskii [11]. Bridges [12] considered the problem of transverse instability of solitary waves for water-wave problem, from both the model equation point of view and the water wave-equations.

In the present paper as a terminology, we call the instability of waveguides the secondary one, because the waveguides themselves are the products of the transverse instability of the carrier wave. The paper organized as follows. In Section 2 the waveguide solutions of Eq. (1.1) are presented, parametrized by the wave number k of the carrier periodic wave, the derivation of the DS equation from (1.1) for slowly varying amplitudes of carrier wave is shortly given, and also the main arguments of the analysis of the transverse instability of the carrier wave are presented. In Section 3 we formulate the results of the linear analysis of the stability of waveguides which is restricted to longitudinal long-wave perturbations. We give the analysis itself, related to our notations, in Appendix A in detail. Section 4 is devoted to description of the results of the numerical stimulation of the nonlinear stage of waveguide instability with respect to arbitrary perturbations. In Section 5 we give our conclusion and discussions.

In the paper by Haragus-Courcelle and Il'ichev [1] the authors missed in the formula (2.8) the term with the third derivative during derivation of Eq. (1.1) for long gravity-flexural waves. We find it expedient to correct this inexactitude here and present the derivation in Appendix B.

2. Wave-guide solutions. Transverse instability of carrier wave

In this section we present the waveguide solution of (1.1) and derive from (1.1) the Davey–Stewartson equations for slowly varying in space and time amplitudes of wavepackets. The results of the carrier wave instability on its nonlinear stage leading to the formation of waveguides are also presented.

We consider solutions to (1.1) of the travelling wave type, propagating with a speed V in the x-axis direction. These solutions $\eta = \eta(x - Vt, y)$ obey the equation

$$\partial_{yy}\eta - V\partial_{\varsigma\varsigma}\eta + \partial_\varsigma(\eta\partial_\varsigma\eta) + \partial_\varsigma^4\eta + \partial_\varsigma^6\eta = 0, \tag{2.1}$$

where $\varsigma = x - Vt$. We assume further that $V = V_1 + \mu$, where $V_1 = -k^2 + k^4$, and $k > 1/\sqrt{5}$ is the real parameter, having the meaning of the wave number [4], and μ is a small parameter. Eq. (2.1) has the waveguide solution [4], at the lowest order in μ given by

$$\eta = a_0^* \cos k(x - Vt), \quad a_0^* = a_0^*(y) = \pm 2k\sqrt{\frac{2\mu}{\chi}}\,\mathrm{sech}\big(k\sqrt{|\mu|}\,y\big), \tag{2.2}$$

where

$$\chi = \frac{1}{6(1-5k^2)} < 0.$$

The form of a waveguide is given in Fig. 1. The wave (2.2) is a subcritical one: its speed V is less than the bifurcation value V_1, i.e., $\mu < 0$.

Let us look for the asymptotic solution of (1.1) in the form

$$\begin{aligned}
&\eta = \epsilon A(T, X, Y)\exp(\mathrm{i}\theta) + \epsilon^2 A_2(T, X, Y)\exp(2\mathrm{i}\theta) + \text{c.c.} + \epsilon^2 A_0(T, X, Y) + \mathrm{O}(\epsilon^3),\\
&\theta = k(x - Vt), \quad V = V_1 - \epsilon^2, \quad T = \epsilon t, \quad X = \epsilon x, \quad Y = \epsilon y,
\end{aligned} \tag{2.3}$$

where ϵ is a small parameter, A and A_2 are slowly varying complex amplitudes, and A_0 is the real function describing the mean flow. Substituting (2.3) in (1.1) and collecting terms at $\epsilon^m \exp(\mathrm{i}n\theta)$, $\{m, n\} = \{1, 1\}, \{2, 1\}, \{3, 1\}, \{2, 2\}, \{4, 0\}$, we get after some algebra the DS equations [4]:

$$\begin{aligned}
&\mathrm{i}A_\tau - kA + \frac{\omega''(k)}{2}A_{XX} - \frac{\chi}{k}A|A|^2 - kAA_0 + \frac{1}{k}A_{YY} = 0,\\
&-\omega'(k)A_{0XX} + |A|^2_{XX} + A_{0YY} = 0,
\end{aligned} \tag{2.4}$$

where $\omega(k) = -k^3 + k^5$, $\tau = \epsilon T$, the subscripts τ, X and Y denote differentiation with respect to corresponding variables, prime denotes differentiation with respect to k, and we keep the old notation X for the combination $X - \omega'(k)T$. Our further analysis of stability concerns Eqs. (2.4).

Eqs. (2.4) have the particular solution $A = \Psi(Y)$, $A_0 = 0$, where $\Psi(Y)$ is a real function obeying the equation

$$\frac{\mathrm{d}^2}{\mathrm{d}Y^2}\Psi(Y) = k^2\Psi(Y) + \chi\Psi^3(Y). \tag{2.5}$$

The localized solution of Eq. (2.5) is given by

$$\Psi(Y) = \pm\sqrt{-\frac{2}{\chi}}\,k\,\mathrm{sech}\,kY. \tag{2.6}$$

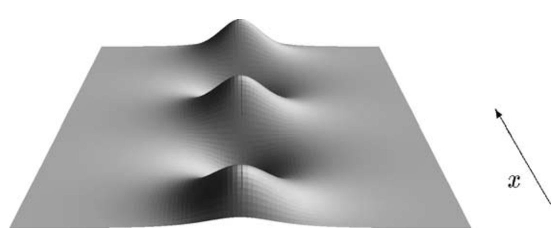

Fig. 1. The form of the waveguide for the surface wave of elevation. Arrow shows the direction of propagation.

We recall that $k > 1/\sqrt{5}$. Comparing (2.2) with (2.3) and (2.6) one gets the correspondence

$$\Psi(Y) = \frac{a_0^*}{2\epsilon}, \quad \epsilon^2 = -\mu.$$

Next we substitute the new variables $a = |A|$, $\psi = \arg A$ into Eqs. (2.4) to get

$$\begin{aligned}
&a_\tau + \frac{\omega''}{2}(2a_X\psi_X + a\psi_{XX}) + \frac{1}{k}(2a_Y\psi_Y + a\psi_{YY}) = 0,\\
&-a\psi_\tau - ka + \frac{\omega''}{2}\left(a_{XX} - a\psi_X^2\right) - \frac{\chi}{k}a^3 - kA_0a + \frac{1}{k}\left(a_{YY} - a\psi_Y^2\right) = 0,\\
&-\omega' A_{0XX} + \left(a^2\right)_{XX} + A_{0YY} = 0.
\end{aligned} \tag{2.7}$$

The solution of (2.7), corresponding to the carrier wave is given by

$$a = a^0 = \text{const}, \quad \psi = \psi^0 = \left(-k - \left(a^0\right)^2\frac{\chi}{k}\right)\tau, \quad A_0 = 0.$$

Substituting

$$a = a^0 + \delta a, \quad \psi = \psi^0 + \delta\psi, \quad A_0 = \delta A_0$$

into (2.7) and assuming

$$\begin{aligned}
&\delta a = \alpha_1 \exp \mathrm{i}(\kappa_\| X + \kappa_\perp Y - \Omega\tau), \qquad \delta\psi = \alpha_2 \exp \mathrm{i}(\kappa_\| X + \kappa_\perp Y - \Omega\tau),\\
&\delta A_0 = \alpha_3 \exp \mathrm{i}(\kappa_\| X + \kappa_\perp Y - \Omega\tau),
\end{aligned}$$

where, α_i, $i = 1, 2, 3$, are constants, one gets the dispersion equation

$$\Omega^2 = \frac{1}{4}\left(\omega''\kappa_\|^2 + 2\frac{\kappa_\perp^2}{k}\right)^2 + \left(a^0\right)^2\left(\frac{k\kappa_\|^2}{\omega'\kappa_\|^2 - \kappa_\perp^2} + \frac{\chi}{k}\right)\left(\omega''\kappa_\|^2 + 2\frac{\kappa_\perp^2}{k}\right). \tag{2.8}$$

Consider next the homogeneous transverse perturbations, when $\kappa_\| = 0$. The dispersion equation (2.8) then takes the form

$$\Omega^2 = \frac{\kappa_\perp^4}{k^2} + 2\frac{(a^0)^2}{k^2}\chi\kappa_\perp^2. \tag{2.9}$$

The wavenumbers $\kappa_\perp$, satisfying (2.9) and lying inside the interval $(0, \kappa_0)$, where $\kappa_0^2 = -2(a^0)^2\chi$, correspond to growing with time perturbations. The value κ_0 gives the threshold of instability, i.e., for all $\kappa_\perp \geqslant \kappa_0$, the carrier periodic wave remains stable. From (2.9) one gets the maximal growth rate: $\Omega(\kappa_{\max}) = -\mathrm{i}(a^0)^2\chi/k$, $\kappa_{\max}^2 = -(a^0)^2\chi$, $\kappa_{\max} < \kappa_0$.

On the nonlinear stage of instability, the carrier wave is subjected to self-channeling, disintegrating to waveguides, similar to (2.2) ([4], see also [13]).

3. Secondary instability of the waveguide

In this section we formulate the results of the linear analysis of the stability of waveguides with respect to longitudinal long-wave perturbations.

We make the substitution $B = \sqrt{|\chi|}\,A$ in the first equation (2.4). Then the soliton solution (2.6) transforms into

$$\Phi(Y) = \pm\sqrt{2}\,k\,\mathrm{sech}(kY).$$

Let us put $B = \Phi(Y) + u + \mathrm{i}v$, $A_0 = w$, where u, v, w are small perturbations (real functions), depending on τ, X, Y. Neglecting nonlinear terms in (2.4), one gets

$$\begin{aligned}
&\left(\frac{\omega''(k)k}{2}\partial_X^2 + \partial_Y^2 + 3\Phi^2(Y) - k^2\right)u - k^2\Phi(Y)w = k\partial_\tau v,\\
&\left(\frac{\omega''(k)k}{2}\partial_X^2 + \partial_Y^2 + \Phi^2(Y) - k^2\right)v = -k\partial_\tau u,\\
&\left[-\omega'(k)\partial_X^2 + \partial_Y^2\right]w + \frac{2}{|\chi|}\Phi(Y)\partial_X^2 u = 0.
\end{aligned} \tag{3.1}$$

We consider here the long-wave longitudinal perturbations, i.e., the functions in (3.1) are looked for in the form

$$\{u, v, w\} = \{u^*, v^*, w^*\}\exp\left(\frac{\varepsilon\lambda\tau}{k}\right)\exp(\mathrm{i}\varepsilon l X) + \text{c.c.}, \quad \varepsilon \ll 1, \tag{3.2}$$

where $u^*,\ v^*,\ w^*$ are real functions depending only on Y. Substituting (3.2) in (3.1), we get

$$\begin{aligned}
&L_+u^* = \varepsilon^2 s u^* + k^2\Phi(Y)w^* + \varepsilon\lambda v^*,\\
&L_-v^* = \varepsilon^2 s v^* - \varepsilon\lambda u^*,\\
&Sw^* = \varepsilon^2\delta\Phi(Y)u^*,\\
&L_+ = \frac{\mathrm{d}^2}{\mathrm{d}Y^2} + 3\Phi^2(Y) - k^2, \quad L_- = \frac{\mathrm{d}^2}{\mathrm{d}Y^2} + \Phi^2(Y) - k^2, \quad S = \frac{\mathrm{d}^2}{\mathrm{d}Y^2} + \varepsilon^2\omega'(k)l^2,
\end{aligned} \tag{3.3}$$

where

$$\delta = \frac{2l^2}{|\chi|}, \qquad s = \frac{\omega''(k)kl^2}{2}. \tag{3.4}$$

The first two equations in (3.3) determine the spectral problem under the constraint, given by the third equation. Instability takes place if there exists a vector eigenfunction $\{u^*, v^*, w^*\}^{\mathrm{T}}$, called unstable, obeying (3.3) for some $\lambda > 0$. The components u^* and v^* of this eigenfunction must tend to zero at infinity. The component w^* must also tend to zero for $|y| \to \infty$ when the operator S is an elliptic one, and have the periodic asymptotic at infinity when S is the hyperbolic operator. The construction of the unstable eigenfunction is achieved by expanding $u^*,\ v^*,\ w^*$ and λ in power series in ε (see (A.1)) and further recurrent calculation of the expansion terms. The detailed construction of the unstable eigenfunction is given in Appendix A.

We get for the growth rate λ_0, being the first term in the expansion in ε of λ:

$$\lambda_0^2 = \frac{4}{3}\left(\delta k^4 - sk^2\right), \qquad \lambda_0^2 = 4sk^2, \tag{3.5}$$

or

$$\lambda_0^2 = \frac{4}{3}l^2k^4\left(\frac{1}{3|1-5k^2|} - 10k^2 + 3\right), \qquad \lambda_0^2 = l^2k^4\left(-3 + 10k^2\right).$$

Eqs. (3.5) correspond to even and odd perturbations of wave field B, respectively. Their behaviour, as determined by Eqs. (3.5), depends on the sign of the coefficient s which, in its turn, depends on the location of the wave number k of the carrier wave on the real axis.

First, let us consider the case $\omega'(k) < 0$. For $s > 0$ ($\omega''(k) > 0$), $k \in I_1$, where

$$I_1 = \left\{k \in \mathbb{R}, \sqrt{\frac{3}{10}} < k < \sqrt{\frac{3}{5}}\right\}.$$

It is seen from (3.5) that the perturbations with the even neutral eigenfunction $v_0 \neq 0$ (see Appendix A) grow with time and the instability of waveguide with respect to the even perturbations takes place. It follows from (3.4), (3.5) that for $k \in I_1' \subset I_1$,

$$I_1' = \left\{k \in \mathbb{R}, \sqrt{\frac{3}{10}} < k < \frac{\sqrt{225 + 15\sqrt{33}}}{30} \approx 0.588\right\}.$$

the right-hand side of the first equation in (3.5) is positive also at $s > 0$. Consequently, the perturbations with the odd neutral eigenfunction $u_0 \neq 0$ also grow and, hence, inside the subinterval I_1' the waveguide is unstable with respect to both type of perturbations (unlike the case of the Schrödinger equation, where instability takes place only with respect to even perturbations everywhere inside I_1). The DS equations for $k \in I_1$ have the elliptic–elliptic type, i.e., the spatial differential operators at A and A_0 in (2.4) are both elliptic.

For $\omega''(k) < 0$, i.e., at $k \in I_2$, where

$$I_2 = \left\{k \in \mathbb{R},\ \sqrt{\frac{1}{5}} < k < \sqrt{\frac{3}{10}}\right\},$$

only odd perturbation (corresponding to $u_0 \neq 0$) grows. In this case the type of the system (2.4) is a hyperbolic-elliptic one, i.e., the corresponding operator in the first equation in (2.4) has the hyperbolic type and in the second equation it has the elliptic one.

The eigenfunction w^* has the periodic asymptotic at infinity, and the waveguide is unstable with respect to both types of perturbations for $\omega''(k) > 0$ and $\omega'(k) > 0$, i. e., at $k \in I_3$,

$$I_3 = \left\{ k \in \mathbb{R},\ k > \sqrt{\frac{3}{5}} \right\},$$

when the type of the system of Eqs. (2.4) is the elliptic-hyperbolic one. The hyperbolic–hyperbolic type of (2.4) is impossible, because it is impossible to satisfy the both inequalities $\omega''(k) < 0$ and $\omega'(k) > 0$ simultaneously.

4. Nonlinear stage of secondary instability

In this section we present the results of the numerical analysis of the nonlinear stage of waveguide instability with respect to arbitrary perturbations.

The DS equations were solved for elliptic–elliptic, hyperbolic–elliptic and elliptic–hyperbolic cases. The initial data for all these calculations were taken in the form of the perturbed waveguide by local small-amplitude disturbance. Both symmetric and asymmetric in the Y-direction disturbances were examined. Waveguides are subjected to instability in all cases.

Fig. 2 shows the form of the developed instability for the elliptic–elliptic case of the DS equations ($k \in I_1$). In this case the instability results in the local collapse of the waveguide, accompanying the collapse of the mean flow. For the sake of comparison we present in Fig. 3 the picture of the development of the instability for the fluid of the infinite depth as governed by the elliptic Schrödinger equation.

For the hyperbolic–elliptic case the waveguide is subjected to slow "snake" instability which is similar to that one, governed by the hyperbolic Schrödinger equation [10]. The mean flow, generated by perturbations of the waveguide disintegrates with time into smaller structures (Fig. 4).

For the case of the elliptic–hyperbolic DS equation the solution for the mean flow is non-local as shown in Fig. 5. In this case waves move with large enough speed and we see some idealization of real physical process similar to effects in supersonic gas dynamics or shallow water.

The explicit three-layer cross type scheme was used for the approximation of wave amplitude equation:

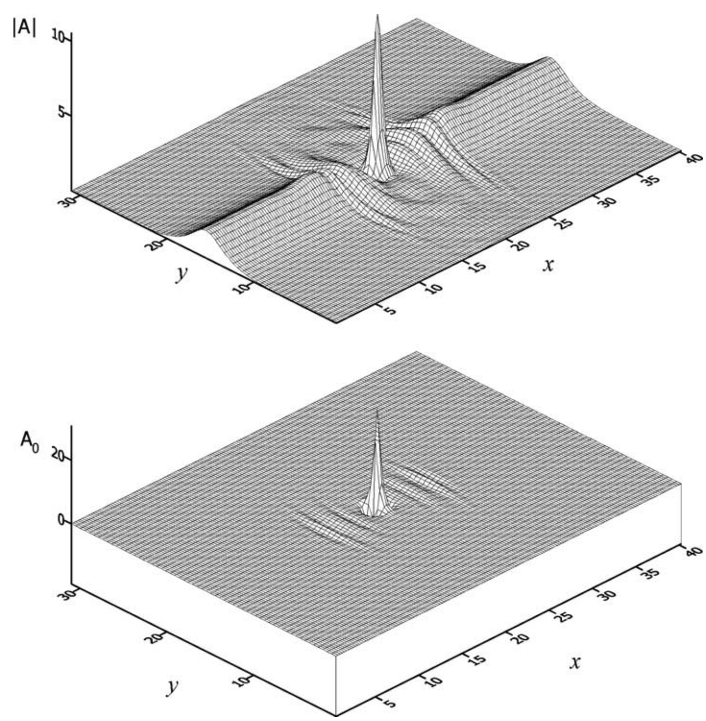

Fig. 2. Collapse of the waveguide amplitude and of the mean flow for $k \in I_1$.

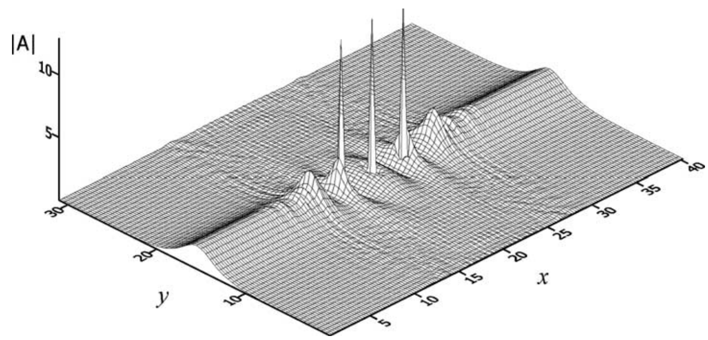

Fig. 3. Collapse of the waveguide amplitude for the fluid of infinite depth.

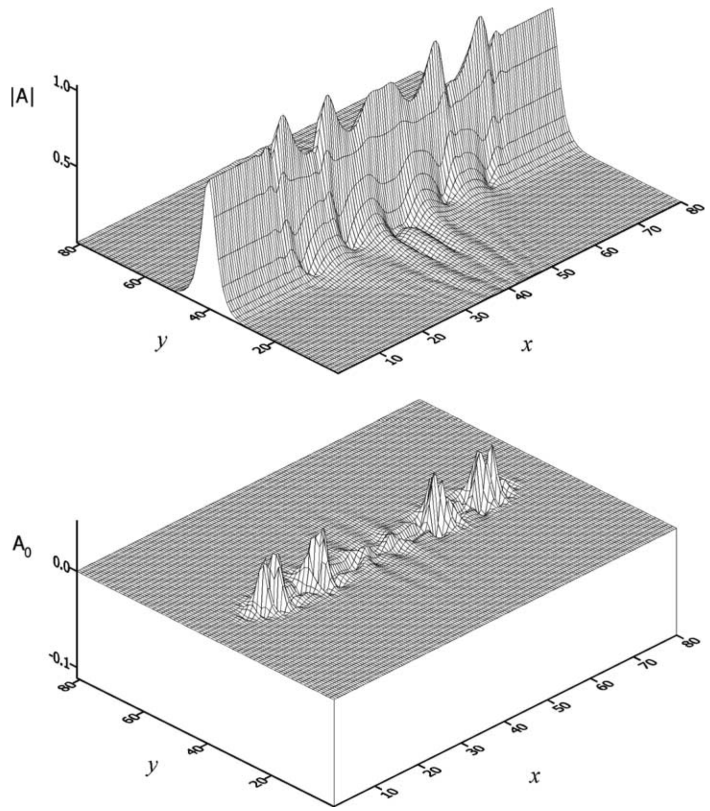

Fig. 4. Snake instability of the waveguide and disintegration of the mean flow for $k \in I_2$.

$$\mathrm{i}\frac{A_{i,j}^{n+1} - A_{i,j}^{n-1}}{2\Delta\tau} - kA_{i,j}^{n} + \frac{\omega''(k)}{2}\frac{A_{i+1,j}^{n} + A_{i-1,j}^{n} - 2A_{i,j}^{n}}{\Delta x^2} - \frac{\chi}{k}A_{i,j}^{n}\left|A_{i,j}^{n}\right|^2 - kA_{i,j}^{n}A_{0i,j}^{n}$$
$$+ \frac{1}{k}\frac{A_{i,j+1}^{n} + A_{i,j-1}^{n} - 2A_{i,j}^{n}}{\Delta y^2} = 0.$$

The condition of stability $\Delta t < c\min(\Delta x^2, \Delta y^2)$ (usually used for nonstationary Schrödinger equation) puts the constraint on the temporal step in the scheme. The value of c is obtained from numerical experiment.

If amplitude for the time step n is known then the mean flow for this step can be calculated independently. For the calculation of flows the following scheme was applied:

$$-\omega'(k)\frac{A_{0i+1,j} + A_{0i-1,j} - 2A_{0i,j}}{\Delta x^2} + \frac{|A^2|_{i+1,j} + |A^2|_{i-1,j} - 2|A^2|_{i,j}}{\Delta x^2} + \frac{A_{0i,j+1} + A_{0i,j-1} - 2A_{0i,j}}{\Delta y^2} = 0.$$

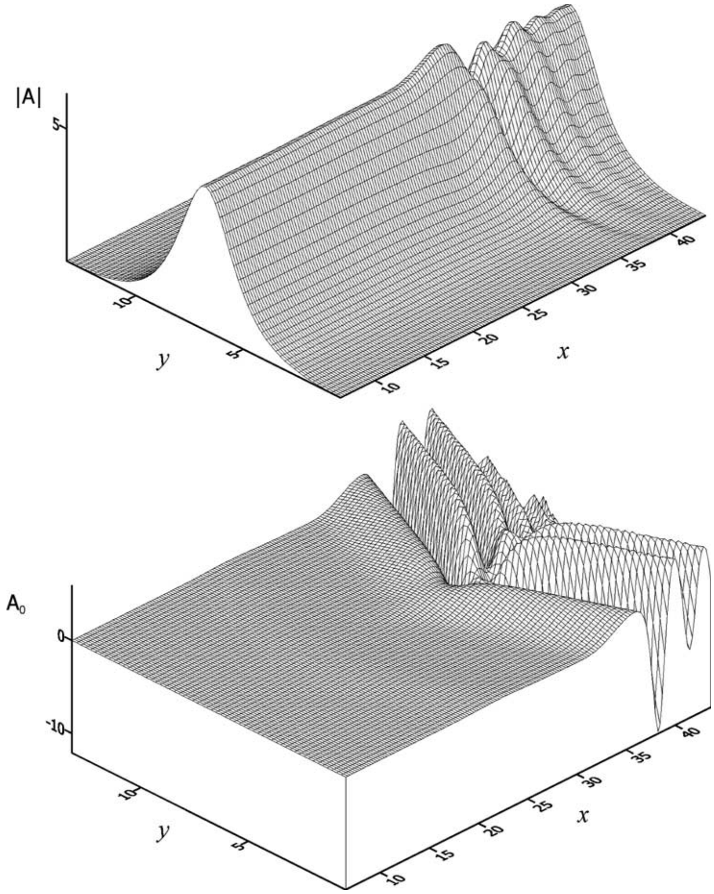

Fig. 5. Development of instability for $k \in I_3$.

Equations are solved for some rectangle $(X, Y) \in \{0, X_{\max}\} \times \{Y_{\min}, Y_{\max}\}$. For the case of the hyperbolic mean flow we have two boundary conditions at the boundary $X = 0$ and one condition at the boundaries $Y = Y_{\min}$ and $Y = Y_{\max}$. All conditions are posed to be compatible with the waveguide solution. Boundaries are placed far away from the spatial domain under investigation to exclude interference with reflected waves. We note, that by the choice of boundary $X = 0$ for posing conditions (rather than the boundary $X = X_{\max}$) we assume that all the waves involved into the process propagate in the direction opposite to that one of the X-axis.

We can solve corresponding system of difference equations explicitly. Values for $i + 1$ spatial step are obtained from values of $i - 1$ and i steps. We use the scheme stability condition $\Delta y/\Delta x > C$, where C is the characteristic speed. To calculate flows in the case of hyperbolic mean flow, the explicit d'Alembert formula was applied as the alternative way of calculation. The effective numerical algorithm of calculation of corresponding integrals is equivalent to the method of characteristics and requires condition $\Delta y/\Delta x = C$.

For the calculation of elliptic flows we pose one boundary condition on all boundaries of the region. This leads to the system of implicit difference equations for the mean flow. The solution of this system was interpreted as stationary solution of some thermal conductivity equation. Iteration process based on the absolutely stable three-layer numerical scheme was used to get this solution.

5. Conclusion and discussion

The monochromatic carrier wave on the surface of the perfect fluid of finite depth under the elastic ice-plate is unstable with respect to transverse perturbations. This instability results in formation of waveguides. The waveguide is itself unstable and the character of its instability depends on the wavenumber k of the carrier wave.

In case when $k \in I_1 = (\sqrt{3/10}, \sqrt{3/5})$ the waveguide is found to be subjected to the local collapse, which differs from that one in the fluid of infinite depth. In the latter case the waveguide first disintegrates in longitudinal direction to localized wave clots which are then collapse (Fig. 3).

In the case of the finite depth fluid, treated here, only the central part of the waveguide collapses along with the mean flow (Fig. 2). The process of collapse in this case can be thought as the nonlinear resonance between the waveguide and the mean flow: perturbations of the central part of the waveguide are excited by the mean flow, while the instability of the waveguide periphery is damped. For elliptic–elliptic DS equation, which is asymptotically equivalent to the governing Eq. (1.1) and according to this reason is assumed here to support the development of corresponding instabilities, the localized wave is known to collapse at a finite time [14,15].

It follows from the dipersion relation (2.8) that also longitudinal self-focusing of the carrier wave takes place for $k \in I_1$. This instability is governed by the "embedded" in (1.1) Kawahara equation for plane waves

$$\partial_t \eta + \eta \partial_x \eta + \partial_x^3 \eta + \partial_x^5 \eta = 0.$$

Self-focusing leads to the decay of the carrier wave into the sequence of the envelope solitary waves for $k \in I_1 \cup (\sqrt{3}/5, 1/\sqrt{5})$ [16]. This decay takes place under action of longitudinal perturbations ($\kappa_\perp = 0$ in (2.8)) for $\kappa_\| \in (0, \kappa_1)$, where

$$\kappa_1^2 = -\frac{(a^0)^2(25k^2 - 3)}{3k^2(10k^2 - 3)(5k^2 - 3)(5k^2 - 1)}.$$

In this context, the carrier wave with $k \in I_1$, coming from the free surface under the ice cover is subjected to at least two competing instability mechanizms, which both lead to collapse, and in this range of wave numbers the waveguide structures may not appear at all if longitudinal self-focusing predominates.

In the case $k \in I_2 = (\sqrt{1/5}, \sqrt{3/10})$ the waveguide instability qualitatively the same as for the fluid of infinite depth [10]. The mean flow is disintegrated to the smaller structures (Fig. 4). The longitudinal instability of the carrier wave is absent in this case, and the waveguide structures may be expected, when the carrier wave propagates under the ice cover.

For $k \in I_3 = (\sqrt{3/5}, \infty)$, the DS equation has elliptic–hyperbolic type and, therefore, the local perturbation of the waveguide generates the non-localized perturbations of the mean flow, which grow rapidly. The rate of growth of these perturbations is large enough to leave behind the corresponding rate of decay of the waveguide. As a consequence, the results of the numerical calculations, presented in Fig. 5 do not show exactly the character of the development of instability of waveguide, though it can be seen that the waveguide is disintegrated to the localized structures. The localized solutions of the elliptic–hyperbolic DS are known to be subjected to collapse [17].

For this range of wavenumbers of the carrier wave the localized perturbation of the waveguide originates the mean flow in the whole space. The physical meaning of such wave solutions is not clear.

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Appendix A. Construction of unstable eigenfunctions

Following Zakharov and Rubenchik [5], Ablowitz and Segur [7] we look for solutions of (3.3) in the form of the formal asymptotic series expansion:

$$u^* = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots, \qquad v^* = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots,$$
$$w^* = \varepsilon^2 w_0 + \cdots, \qquad \lambda = \lambda_0 + \varepsilon^2 \lambda_1 + \cdots. \tag{A.1}$$

Substituting (A.1) into (3.3), one gets in the zeroth order in ε

$$L_+ u_0 = 0, \qquad L_- v_0 = 0,$$

which implies that nonzero solution has the form

$$u_0 = \dot{\Phi}(Y), \qquad v_0 = \Phi(Y). \tag{A.2}$$

In the first order in ε one has

$$L_+ u_1 = \lambda_0 v_0, \qquad L_- v_1 = -\lambda_0 u_0.$$

This system of equations has a solution

$$u_1 = \lambda_0 \frac{\mathrm{d}\Phi}{\mathrm{d}\alpha}, \quad v_1 = -\frac{\lambda_0}{2} Y\Phi(Y), \quad \alpha = k^2. \tag{A.3}$$

Finally, in the second order in ε we get:

$$L_+ u_2 = \lambda_0 v_1 + s u_0 + \alpha\Phi(Y) w_0, \qquad L_- v_2 = -\lambda_0 u_1 + s v_0, \qquad S w_0 = \delta\Phi(Y)(u_0 + \varepsilon u_1). \tag{A.4}$$

The solvability condition for the first two equations in (A.4) yields the orthogonality of right-hand sides to u_0 and v_0 from (A.2), respectively. Let us note, that formally in the third equation (A.4) one should write $S_0 = \mathrm{d}^2/\mathrm{d}Y^2$ instead of S, and also omit the term, proportional to ε in the right-hand side of this equation. Yet, firstly, the C-norm of the difference of solutions of $Sf_1 = g$ and $S_0 f_2 = g$, where g is decaying function, is not a value of order ε^2 and it does not depend on ε at all. Secondly, as it will follow from the further analysis, the amplitude of f_1 for g finite and even have the order ε^{-1}, therefore the terms, proportional to ε, have to be also remained in the third equation in (A.4).

For computations of other terms in the expansion (A.1) one needs to compute the value λ_n from the compatibility conditions for the equations for $u_n,\ v_n$. These conditions operate with functions $u_i,\ v_i,\ w_i,\ i \leqslant n-1$, from previous steps. The functions $u_{n-1},\ v_{n-1}$ are determined from the previous steps uniquely due to the invertibility of the operators $L_\pm$ in $L_2(\mathbb{R})$. The same is true for w_{n-1} and S in the case when $\omega'(k) < 0$. If $\omega'(k) > 0$ the right-hand sides of the equations $L_+ u_{n-1} = f_{n-2}$, $L_- v_{n-1} = g_{n-2}$ are still in $L_2(\mathbb{R})$, and therefore u_{n-1} and v_{n-1} also.

The solvability condition of the first two equations in (A.4) has the form

$$\int_{-\infty}^{\infty} u_0\big(\lambda_0 v_1 + s u_0 + \alpha\Phi(Y) w_0\big)\,\mathrm{d}Y = 0, \qquad \int_{-\infty}^{\infty} v_0(-\lambda_0 u_1 + s v_0)\,\mathrm{d}Y = 0. \tag{A.5}$$

The quantity λ_0 is determined from these conditions. The eigenfunctions $u_0,\ v_0,\ u_1,\ v_1$ are determined from (A.2), (A.3), respectively. The function w_0 is obtained by getting inverse of the operator S in (A.4).

Let us consider the procedure of construction of w_0 in the case when $\omega'(k) < 0$. This procedure can be applied analogously in the hyperbolic case ($\omega'(k) > 0$), though w_0 is not decaying any more in this case. We look for the solution of the third equation of (A.4) in the form $w_0 = w_0^0 + \varepsilon w_0^1$ and consider the following two equations separately

$$S w_0^0 = \delta\Phi(Y) u_0, \qquad S w_0^1 = \delta\Phi(Y) u_1. \tag{A.6}$$

Let us denote $F[f]$ the Fourier transform of f:

$$F[f] = \frac{1}{2\pi} \int_{-\infty}^{\infty} f \exp(-\mathrm{i}\nu x)\,\mathrm{d}x.$$

Applying the Fourier transformation to both sides of the first equation in (A.6) and changing the order of differentiation in Y in the definite integral, one gets

$$F\big[w_0^0\big] = -\frac{\mathrm{i}\delta}{2} \frac{\nu^2 \operatorname{cosech}\vartheta}{\nu^2 + \varepsilon^2 l^2 |\omega'(k)|}, \quad \vartheta = \left(\frac{\pi\nu}{2\sqrt{\alpha}}\right). \tag{A.7}$$

To get (A.7) we use the formula

$$F\big[\operatorname{sech}^2(x)\big] = \frac{\nu}{2} \operatorname{cosech}\left(\frac{\pi\nu}{2}\right).$$

Applying the inverse Fourier transformation to (A.7) we get

$$w_0^0 = -\frac{\mathrm{i}\delta}{2} \int_{-\infty}^{\infty} \frac{\nu^2 \operatorname{cosech}\vartheta}{\nu^2 - \varepsilon^2 l^2 \omega'(k)} \exp(\mathrm{i}\nu Y)\,\mathrm{d}\nu. \tag{A.8}$$

The integrand in (A.8) has simple poles on the imaginary axis in the complex ν-plane. The principal part of the assymptotic of the integral (A.8) on the positive infinity $Y > 0$ can be computed from the following arguments. Let us consider the contour in the complex ν-plane, consisting from the real axis and contour $\mathcal{C}^+$, to be passed from right to left and lying above the pole $\nu_0^+ = \mathrm{i}\varepsilon l\sqrt{|\omega'(k)|}$ (Fig. 6). From the Liouville residue theorem one has

$$w_0^0 + \int_{\mathcal{C}^+} F\big[w_0^0\big] \exp(\mathrm{i}\nu Y)\,\mathrm{d}Y = 2\pi\mathrm{i}\,\mathrm{res}_{\nu_0^+} F\big[w_0^0\big] \exp(\mathrm{i}\nu) Y. \tag{A.9}$$

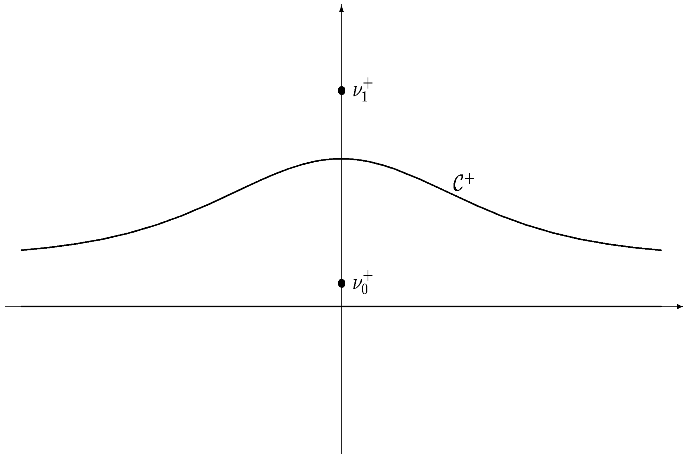

Fig. 6. Form of the contour $\mathcal{C}^+$ in the complex ν-plane.

Using the fact that the integral along $\mathcal{C}^+$ tends to zero at plus infinity faster than the residue at the right-hand side of (A.9) we neglect it in getting the principal part of the asymptotic:

$$w_0^0 \to \delta\sqrt{\alpha}\,\exp\bigl(-\varepsilon l\sqrt{\left|\omega'(k)\right|}\,Y\bigr), \quad Y \to +\infty. \tag{A.10}$$

The asymptotic at minus infinity can be computed analogously, using the contour $\mathcal{C}^-$ which is symmetric to $\mathcal{C}^+$ about the real axis:

$$w_0^0 \to -\delta\sqrt{\alpha}\,\exp\bigl(-\varepsilon l\sqrt{\left|\omega'(k)\right|}\,Y\bigr), \quad Y \to -\infty. \tag{A.11}$$

It can be easily seen that the function w_0^0 is an odd one.

The principal part of the asymptotic at infinity of the even function w_0^1 is computed analogously:

$$w_0^1 = -\frac{\pi\lambda_0\delta}{8\alpha\sqrt{\alpha}}\int_{-\infty}^{\infty}\frac{\nu^2\,\mathrm{cosech}^2\,\vartheta\,\cosh\vartheta}{\nu^2-\varepsilon^2 l^2\omega'(k)}\exp(\mathrm{i}\nu Y)\,\mathrm{d}\nu.$$

This function obeys the second equation (A.6) and it has the form:

$$w_0^1 \to -\frac{1}{2\varepsilon}\frac{\lambda_0\delta}{\sqrt{\alpha}l\sqrt{|\omega'(k)|}}\exp\bigl(-\varepsilon l\sqrt{\left|\omega'(k)\right|}\,|Y|\bigr), \quad Y \to \pm\infty. \tag{A.12}$$

In can be seen from (A.12) that the amplitude w_0^1 has the order ε^{-1} and as a consequence, it is necessary to take into account the term, proportional to u_1 in the right-hand side of the third equation in (A.4). The function w_0^1, being an even one, has no influence on the solvability condition (A.5); these conditions contain only the odd part w_0^0 of the function w_0.

To compute the integrals in $(A.5)$ in zeroth order in ε it should be noted that w_0^0 tends to the solution of the equation

$$S_0 w_{00}^0 = \delta\Phi(Y)u_0 \tag{A.13}$$

in distributional sense for $\varepsilon \to 0$. In other words

$$\int_{-\infty}^{\infty}\bigl(w_0^0 - w_{00}^0\bigr)\varphi \to 0, \quad \varepsilon \to 0,$$

where φ is an arbitrary smooth rapidly decreasing function on the real axis $\mathbb{R}$.

It follows then that at zeroth order in ε the function w_{00}^0 may be written in (A.5) instead of w_0. The solution of Eq. (A.13) is given by

$$w_{00}^0 = \sqrt{\alpha}\delta\tanh(\sqrt{\alpha}Y). \tag{A.14}$$

It can be easily seen, that (A.14) has the asymptotics (A.10), (A.11) as $\varepsilon \to 0$.

Substituting (A.14) into (A.5) one gets (3.5) (coming back from α to k in notations).

In the case when S is hyperbolic the eigenfunction w^* is no more decaying but having the periodic asymptotics at infinity. It is constructed analogously to the case treated above by use of the contour $\mathcal{C} = \Gamma_1 \cup \Gamma_2$ where the contour Γ_1 lies above the poles of the integrand in (A.8) on the real axis and the contour Γ_2 lies below these poles. The contours $\Gamma_{1,2}$ contribute to the asymptotics at minus and plus infinities, correspondingly.

Appendix B. Derivation of (1.1) for flexural-gravity waves

In this section we derive (1.1) for long surface flexural-gravity waves of small amplitude in the presence of an elastic ice-plate. The ice-plate is assumed to obey the equations of the theory of thin plates [18].

The Euler system with corresponding additional surface pressure has the form (subscripts denote differentiation with respect to the corresponding variables)

$$\begin{aligned}
&\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0, \quad -H < z < \eta(x, y, t),\\
&\varphi_z = 0, \quad z = -H,\\
&\eta_t + \eta_x\varphi_x + \eta_y\varphi_y = \varphi_z, \quad z = \eta(x, y, t),\\
&\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2 + \varphi_z^2\right) + g\eta + A\eta_{tt} + B\Delta_{xy}^2\eta = 0, \quad z = \eta(x, y, t).
\end{aligned} \tag{B.1}$$

Here $\Delta_{xy} = \partial_{xx}^2 + \partial_{yy}^2$, φ is the velocity potential, η is the liquids surface deviation from equilibrium $z = 0$, x denotes the horizontal unbounded variable, and $A = \rho_i h/\rho_w$, $B = Eh^3/[12\rho_w(1 - \nu_0^2)]$, where ρ_i and ρ_w are the ice and water densities, h and H is the ice thickness and water depth, E is the Young module and ν_0 is the Poisson ratio of the ice. We suppress in (B.1) the nonlinear terms corresponding to the additional pressure caused by the elastic plate, because they give no contribution in the equation we derive.

In order to determine the relative importance of different terms in the equations above we introduce the following small parameters

$$\varepsilon = \frac{a}{H}, \quad \gamma = \frac{B}{g\lambda^4}, \quad \delta = \frac{AH}{\lambda^2}, \quad \epsilon = \frac{H^2}{\lambda^2},$$

where λ is the characteristic wavelength, and a is the characteristic wave amplitude. The following dimensionless variables can be defined:

$$t' = \frac{(gH)^{1/2}t}{\lambda}, \quad \varphi' = \frac{(gH)^{1/2}\varphi}{ga\lambda}, \quad \eta' = \frac{\eta}{a}, \quad x' = \frac{x}{\lambda}, \quad y' = \frac{y}{\lambda}, \quad z' = \frac{z}{H}.$$

Then (B.1) is rewritten as follows (omitting the primes):

$$\begin{aligned}
&\epsilon(\varphi_{xx} + \varphi_{yy}) + \varphi_{zz} = 0, \quad -1 < z < \varepsilon\eta,\\
&\varphi_z = 0, \quad z = -1,\\
&\eta_t + \varepsilon\eta_x\varphi_x + \varepsilon\eta_y\varphi_y = \epsilon^{-1}\varphi_z, \quad z = \varepsilon\eta,\\
&\varphi_t + \tfrac{1}{2}\varepsilon\left(\varphi_x^2 + \varphi_y^2 + \epsilon^{-1}\varphi_z^2\right) + \eta + \delta\eta_{tt} + \gamma\Delta_{xy}^2\eta = 0, \quad z = \varepsilon\eta.
\end{aligned} \tag{B.2}$$

The velocity potential can be expanded with respect to the vertical coordinate z:

$$\varphi = \varphi^0 + z\varphi_z^0 + \frac{1}{2}z^2\varphi_{zz}^0 + \frac{1}{6}z^3\varphi_{zzz}^0 + \frac{1}{24}z^4\varphi_{zzzz}^0 + \frac{1}{120}z^5\varphi_{zzzzz}^0 + \frac{1}{720}z^6\varphi_{zzzzzz}^0 + \cdots. \tag{B.3}$$

From the first equation in (B.2) we obtain

$$\begin{aligned}
&\varphi_{zz}^0 = -\epsilon\left(\varphi_{xx}^0 + \varphi_{yy}^0\right), \qquad \varphi_{zzz}^0 = -\epsilon\left[\left(\varphi_z^0\right)_{xx} + \left(\varphi_z^0\right)_{yy}\right],\\
&\varphi_{zzzz}^0 = \epsilon^2\left(\varphi_{xxxx}^0 + 2\varphi_{xxyy}^0 + \varphi_{yyyy}^0\right),\\
&\varphi_{zzzzz}^0 = \epsilon^2\left[\left(\varphi_z^0\right)_{xxxx} + 2\left(\varphi_z^0\right)_{xxyy} + \left(\varphi_z^0\right)_{yyyy}\right],\\
&\varphi_{zzzzzz}^0 = -\epsilon^3\left(\varphi_{xxxxxx}^0 + 3\varphi_{xxxxyy}^0 + 3\varphi_{xxyyyy}^0 + \varphi_{yyyyyy}^0\right).
\end{aligned}$$

Using the second equation in (B.2) (boundary condition at the bottom), (B.3), and applying the expansion series method for small ϵ up to (but not including) terms of order ϵ^4, we derive the expression for φ_z^0:

$$\varphi_z^0 = -\epsilon\Delta_{xy}\varphi^0 - \frac{\epsilon^2}{3}\Delta_{xy}^2\varphi^0 - \frac{2}{15}\epsilon^3\Delta_{xy}^3\varphi^0. \tag{B.4}$$

Substituting (B.4) into the last two equations in (B.2) and neglecting terms of order $\varepsilon\epsilon$ and higher, we deduce (dropping the superscript ^{0}) the system

$$\begin{aligned}&\eta_t+\varepsilon\eta_x\varphi_x+\varepsilon\eta_y\varphi_y+\varepsilon\eta\Delta_{xy}\varphi+\Delta_{xy}\varphi+\frac{\epsilon}{3}\Delta_{xy}^2\varphi+\frac{2}{15}\epsilon^2\Delta_{xy}^2\varphi=0,\\&\varphi_t+\frac{\varepsilon}{2}(\varphi_x^2+\varphi_y^2)+\eta+\delta\eta_{tt}+\gamma\Delta_{xy}^2\eta=0.\end{aligned}\tag{B.5}$$

Next, introduce the unknown functions in the form

$$\begin{aligned}&\eta=\eta_0+\eta_1\epsilon+\eta_2\epsilon^2+\mathrm{O}(\epsilon^3),\\&\varphi=\varphi_0+\varphi_1\epsilon+\varphi_2\epsilon^2+\mathrm{O}(\epsilon^3).\end{aligned}\tag{B.6}$$

Neglecting terms of order 3, we look for waves travelling in one direction (to the right), i.e., weakly depending on time in the reference frame moving with the phase speed of the linear wave train having an infinite wavelength:

$$\begin{aligned}&\eta=\hat\eta(\xi,\zeta,\tau,\epsilon),\qquad\hat\varphi(\xi,\zeta,\tau,\epsilon),\\&\xi=x-t,\quad\zeta=\sqrt{\varepsilon}y,\quad\tau=\epsilon^m t,\quad m>0.\end{aligned}$$

Let $m=1$, $\delta=\hat\delta\epsilon$, $\gamma=\hat\gamma\epsilon$, $\beta=0$, $\varepsilon=\hat\varepsilon\epsilon$, where quantities with hat are of order 1, i.e.

$$\frac{E}{12(1-\nu_0^2)\rho_w}\sim\frac{g\lambda^2H^2}{h^3}.$$

In a real media this corresponds to the following values of parameters:

$$h\sim1\ \mathrm{m},\quad E\sim10^9\ \mathrm{N\cdot m^{-2}},\quad H\sim10\ \mathrm{m},\quad\lambda\sim100\ \mathrm{m},\quad a\sim1\ \mathrm{m}.$$

Substituting (B.6) into (B.5), we obtain, up to terms of order ϵ^2,

$$\begin{aligned}&-\hat\eta_{0\xi}-\epsilon\hat\eta_{1\xi}+\epsilon\hat\eta_{0\tau}+\hat\varepsilon\epsilon\hat\eta_{0\xi}\hat\varphi_{0\xi}+\hat\varepsilon\epsilon\hat\eta_0\hat\varphi_{0\xi\xi}+\hat\varphi_{0\xi\xi}+\hat\varepsilon\epsilon\hat\varphi_{0\zeta\zeta}+\epsilon\hat\varphi_{0\xi\xi}+\frac{\epsilon}{3}\hat\varphi_{0\xi\xi\xi\xi}=0,\\&-\hat\varphi_{0\xi}-\epsilon\hat\varphi_{1\xi}+\epsilon\hat\varphi_{0\tau}+\frac12\hat\varepsilon\epsilon\hat\varphi_{0\xi}^2+\hat\eta_0+\epsilon\hat\eta_1+\hat\delta\epsilon\hat\eta_{0\xi\xi}+\hat\gamma\epsilon\hat\eta_{0\xi\xi\xi\xi}=0.\end{aligned}$$

By equating the terms of order 1 and ϵ we find

$$\hat\varphi_{0\xi}=\hat\eta_0,\tag{B.7}$$

and the following equation for $\hat\eta_0$ (by using (B.7)):

$$\left[\hat\eta_{0\tau}+\frac32\hat\varepsilon\hat\eta_0\hat\eta_{0\xi}+\frac12\left(\hat\delta+\frac13\right)\hat\eta_{0\xi\xi\xi}+\frac{\hat\gamma}{2}\hat\eta_{0\xi\xi\xi\xi\xi}\right]_\xi+\frac{\hat\varepsilon}{2}\hat\eta_{0\zeta\zeta}=0.\tag{B.8}$$

Eq. (B.8) can be put in the form (1.1) with the help of scaling transformations.

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